THE EFFECTIVE USE OF BLOCK ALGORITHM FOR MATHEMATICAL TREATMENT OF SOME PROBLEMATIC SYSTEM OF ORDER THREE

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Abstract

The effective block algorithm for mathematical treatment of problematic systems of initial value problem was established and examined in this research. All the conditions are valid when examining the conditions that satisfied the properties of the scheme. The third order problematic differential equations are directly solved and relate with the schemes developed in literature, the results are evidently shows better performance than those researchers in literature.

Keywords: Effective, Block Algorithm, Mathematical Solution, Problematic, Initial Value Problems.

Introduction

This research is based on approximating a class of the problematic system given by

$$y^{d} = f\left(t,\, y(t),\, y'(t),\, \cdots,\, y^{d-1}(t),\, y^{d}(t)\right) \qquad (1)$$

In order for (1) to have a unique solution, certain conditions need to be imposed at the initial point as

$$y^{s}(a) = \eta_s,\, s = 0(1)d-1 \qquad (2)$$

In this research, we proposed the approximation of third order initial value problem where some researcher reduced (1) to ordinary differential equation of first order before approximating it, however, this process can only approximate the mathematical solution one by one and time constraint (Spiegel 1971, Lambert 1973, Awoyemi 1990, Donald, Yusuf & Dominic 2015, Jaun 2002).

Now to avoid these challenges, (Sabo Bakari & Babuba 2021, Raymond, Skwame & Lydia 2021, Kyagya, Raymond & Sabo 2021, Adoghe & Omole 2019, Kuboye, Elusakin & Quadri 2020, Adeyeye & Omar 2019, Sabo, Skwame & Donald 2022) in literature, proposed a method, to solve (1) without reduction process.

The advantage of this research is to address the challenges in predictor-corrector process and reduction process.

(Olabode & Yusuph 2009, Donald *et al.*, 2021, Olabode 2013) proposed schemes for handling (1) straight forward, the process of developing separate predictor is not involved and does not requires more functions to evaluate step by step.

(Spiegel 1971, Awoyemi 1990, Donald, Yusuf & Dominic 2015, Jaun 2002, Sabo Bakari & Babuba 2021, Omar & Suleiman 1999, Sabo, Kyagya & Solomon 2021) later developed block methods for the direct treatment of (1).

Material and Mathematical Algorithm

The algorithm for treating (1) is derived using Linear Block Algorithm (LBA). Theorem one and two showed the stages to develop the block algorithm. The algorithm lead the model of considering the general form of the algorithm while following one by one to yield the expected block algorithm for treating third order initial value problem (Spiegel 1971).

Theorem one is used to yield the block algorithm using

$$y_{n+\xi} = \sum_{i=0}^{2} \frac{(\xi h)^i}{i!} y_n^{(i)} + \sum_{i=0}^{5} \left(\phi_{i\xi} f_{n+i}\right), \quad \xi = m,\, n,\, u,\, v,\, w,\, 1 \qquad (3)$$

Obtain the first and second derivative schemes of the block method from Theorem two as

$$y_{n+\xi}^{(a)} = \sum_{i=0}^{2-a} \frac{(\xi h)^i}{i!} y_n^{(i+a)} + \sum_{i=0}^{5} \Psi_{\xi i a} f_{n+i},\; a = 1_{(\xi=m,\,n,\,u,\,v,\,w,\,1)},\; a = 2_{(\xi=m,\,n,\,u,\,v,\,w,\,1)} \qquad (4)$$

$\phi_{\xi i} = A^{-1}P$ and $\Psi_{\xi i a} = A^{-1}Q$ where

$$A=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{(m)^1}{1!} & \frac{(n)^1}{1!} & \frac{(u)^1}{1!} & \frac{(v)^1}{1!} & \frac{(w)^1}{1!} & \frac{(1)^1}{1!} \\ 0 & \frac{(m)^2}{2!} & \frac{(n)^2}{2!} & \frac{(u)^2}{2!} & \frac{(v)^2}{2!} & \frac{(w)^2}{2!} & \frac{(1)^2}{2!} \\ 0 & \frac{(m)^3}{3!} & \frac{(n)^3}{3!} & \frac{(u)^3}{3!} & \frac{(v)^3}{3!} & \frac{(w)^3}{3!} & \frac{(1)^3}{3!} \\ 0 & \frac{(m)^4}{4!} & \frac{(n)^4}{4!} & \frac{(u)^4}{4!} & \frac{(v)^4}{4!} & \frac{(w)^4}{4!} & \frac{(1)^4}{4!} \\ 0 & \frac{(m)^5}{5!} & \frac{(n)^5}{5!} & \frac{(u)^5}{5!} & \frac{(v)^5}{5!} & \frac{(w)^5}{5!} & \frac{(1)^5}{5!} \\ 0 & \frac{(m)^6}{6!} & \frac{(n)^6}{6!} & \frac{(u)^6}{6!} & \frac{(v)^6}{6!} & \frac{(w)^6}{6!} & \frac{(1)^6}{6!} \end{pmatrix}, P=\begin{pmatrix} \frac{(\xi h)^3}{3!} \\ \frac{(\xi h)^4}{4!} \\ \frac{(\xi h)^5}{5!} \\ \frac{(\xi h)^6}{6!} \\ \frac{(\xi h)^7}{7!} \\ \frac{(\xi h)^8}{8!} \\ \frac{(\xi h)^9}{9!} \end{pmatrix}, Q=\begin{pmatrix} \frac{(\xi h)^{3-a}}{(3-a)!} \\ \frac{(\xi h)^{4-a}}{(4-a)!} \\ \frac{(\xi h)^{5-a}}{(5-a)!} \\ \frac{(\xi h)^{6-a}}{(6-a)!} \\ \frac{(\xi h)^{7-a}}{(7-a)!} \\ \frac{(\xi h)^{8-a}}{(8-a)!} \\ \frac{(\xi h)^{9-a}}{(9-a)!} \end{pmatrix}$$

solving the above equation one by one to yield

$\vartheta_s,\ s=0,\ m,\ n,\ u,\ v,\ w,1$

Put the polynomial $x = x_s + th$, in (3) to yield

$$p(x_n th)=\alpha_m y_s+\alpha_n y_s+\alpha_u y_s+ \\ h^3\left(\beta_0 f_s+\beta_m f_{s+m}+\beta_n f_{s+n}+\beta_u f_{s+u}+\beta_v f_{s+v}+\beta_w f_{s+w}+\beta_1 f_{s+1}\right) \qquad (5)$$

Where

$$\phi_0-\frac{1}{5040mnuvw}\xi^3\begin{pmatrix} 15\xi^5-10\xi^6-24m\xi^4+15m\xi^5-24n\xi^4+15n\xi^5-24u\xi^4+15u\xi^5-24v\xi^4+15v\xi^5 \\ -24w\xi^4+15w\xi^5+42mn\xi^3-24mn\xi^4+42mu\xi^3-24mu\xi^4+42mv\xi^3+42nu\xi^3 \\ -24mv\xi^4-24nu\xi^4+42mw\xi^3+42nv\xi^3-24mw\xi^4-24nv\xi^4+42nv\xi^3-24nw\xi^4 \\ +42uv\xi^3-24uv\xi^4+42uw\xi^3-24uw\xi^4+42vw\xi^3-24vw\xi^4-84mnu\xi^2+42mnu\xi^3 \\ -84mnv\xi^2+42mnv\xi^3-84mnw\xi^2+42mnw\xi^3-84muv\xi^2+42muv\xi^3-84muw\xi^2 \\ -84nuv\xi^2+42muw\xi^3+42nuv\xi^3-84mvw\xi^2+42mnw\xi^3-84nuw\xi^2-42muv\xi^3 \\ +42nuw\xi^3-84nvw\xi^2+42nvw\xi^3-84uvw\xi^2+42uvw\xi^3+210mnuv\xi+210mnuw\xi \\ +210mnvw\xi+210muvw\xi+210nuvw\xi-84mnuw\xi^2-84mnuw\xi^2-84mnvw\xi^2- \\ 84muvw\xi^2-84nuvw\xi^2-840mnuvw+210mnuvw\xi \end{pmatrix}$$

$$\phi_m=\frac{1}{5040}\frac{\xi^4\begin{pmatrix} -14\xi^4+14\xi^5+24n\xi^3-15n\xi^4+24u\xi^3-15u\xi^4+24v\xi^3-15v\xi^4+24w\xi^3-15w\xi^4 \\ -42nu\xi^2+24nu\xi^3-42nv\xi^2+24nv\xi^3-42nw\xi^2+24nw\xi^3-42uv\xi^2+24uv\xi^3 \\ -42uw\xi^2+24uw\xi^3-42vw\xi^2+24vw\xi^3+84nuv\xi+84nuw\xi+84nvw\xi+84uvw\xi \\ -42nuv\xi^2-42nuw\xi^2-42nvw\xi^2-42uvw\xi^2-210nuvw+84nuvw\xi \end{pmatrix}}{m(m-1)(m-w)(m-v)(m-u)(m-n)}$$

$$\phi_n = -\frac{1}{5040u}\frac{\xi^4\begin{pmatrix}-15\xi^4+10\xi^5 24m\xi^3-15m\xi^4+24u\xi^3-15u\xi^4+24v\xi^3-15v\xi^4+24w\xi^3-15w\xi^4\\-42mu\xi^2+24mu\xi^3-42mv\xi^2+24mv\xi^3-42mw\xi^2+24mw\xi^3-42uv\xi^2+24uw\xi^3\\-42vw\xi^2+24vw\xi^3+84muv\xi+84muw\xi+84mvw\xi+84uvw\xi-42muv\xi^2-42muw\xi^2\\-42mvw\xi^2 42uvw\xi^2-210muvw+84muvw\xi\end{pmatrix}}{(n-1)(n-w)(n-v)(n-u)(m-n)}$$

$$\phi_u = \frac{1}{5040u}\frac{\xi^4\begin{pmatrix}-15\xi^4+10\xi^5+24m\xi^3-15m\xi^4+24n\xi^3-15n\xi^4+24v\xi^3-15v\xi^4+24w\xi^3\\-15w\xi^4-42mn\xi^2+24mn\xi^3-42mv\xi^2+24mv\xi^3-42mw\xi^2-42nv\xi^2+\\24mw\xi^3+24nv\xi^3-42nw\xi^2+24nw\xi^3-42vw\xi^2+24vw\xi^3 84mnv\xi+84mnw\xi\\+84mvw\xi+84nvw\xi-42mnv\xi^2-42mnw\xi^2-42mvw\xi^2-42nvw\xi^2-210mnvw\\+84mnvw\xi\end{pmatrix}}{(u-1)(u-w)(u-v)(n-u)(m-u)}$$

$$\phi_v = -\frac{1}{5040v}\frac{\xi^4\begin{pmatrix}-15\xi^4+10\xi^5+24m\xi^3-15m\xi^4+24n\xi^3-15n\xi^4+24u\xi^3-15u\xi^4\\+24w\xi^3-15w\xi^4-42mn\xi^2+24mn\xi^3-42mu\xi^2+24mu\xi^3-42nu\xi^2+24nu\xi^3\\-42mw\xi^2+24mw\xi^3-42nw\xi^2+24nw\xi^3-42uw\xi^2+24uw\xi^3+84mnu\xi\\+84mnw\xi+84muw\xi+84nuw\xi-42mnu\xi-42mnw\xi^2-42muw\xi^2-42nuw\xi^2\\-210mnuw+84mnuw\xi\end{pmatrix}}{(v-1)(v-w)(u-v)(n-v)(m-v)}$$

$$\phi_w = -\frac{1}{5040w}\frac{\xi^4\begin{pmatrix}-15\xi^4+10\xi^5+24m\xi^3-15m\xi^4+24n\xi^3-15n\xi^4+24u\xi^3-15u\xi^4+24v\xi^3-15v\xi^4\\-42mn\xi^2+24mn\xi^3-42mu\xi^2+24mu\xi^3-84mnu\xi+84mnv\xi+84nuv\xi^2-42mnv\xi^2\\-42muv\xi^2-42nuv\xi^2-210mnuv+84mnuv\xi\end{pmatrix}}{(w-1)(v-w)(u-w)(n-w)(m-w)}$$

$$\phi_1 = -\frac{1}{5040}\frac{\xi^4\begin{pmatrix}-10\xi^5+15m\xi^4+15n\xi^4+15u\xi^4+15v\xi^4+15w\xi^4-24mn\xi^3-24mu\xi^3\\-24mv\xi^3-24nu\xi^3-24mw\xi^3-24nv\xi^3-24nw\xi^3-24uv\xi^3-24uw\xi^3\\-24vw\xi^3+42mnu\xi^2+42mnv\xi^2+42mnw\xi^2+42muv\xi^2+42muw+42nuv\xi^2\\+42mvw\xi^2+42nuw\xi^2+42nvw\xi^2+42uvw\xi^2-84mnuv\xi-84mnvw\xi-84muvw\xi\\-84nuvw\xi+210mnuvw\end{pmatrix}}{(w-1)(v-1)(u-1)(n-1)(m-1)}$$

Evaluating the schemes of (5) at all points we obtain equations (6), (7) and (8) as shown in tables below

Table 1. Coefficient of the first scheme of (5) which evaluated at all points as

τ_n	y_n	y'_n	y''_n	f_n	$f_{n+\frac{4}{9}}$	$f_{n+\frac{5}{9}}$	$f_{n+\frac{2}{3}}$	$f_{n+\frac{7}{9}}$	$f_{n+\frac{8}{9}}$	f_{n+1}
$\tau_{n+\frac{4}{9}}$	1	$\frac{4}{9}$	$\frac{8}{81}$	$\frac{1398676}{217005075}$	$\frac{500368}{3444525}$	$-\frac{1474432}{3444525}$	$\frac{167488}{295245}$	$-\frac{1948928}{4822335}$	$\frac{104276}{688905}$	$-\frac{729472}{31000725}$
$\tau_{n+\frac{5}{9}}$	1	$\frac{5}{9}$	$\frac{25}{162}$	$\frac{96218375}{8888527872}$	$\frac{19331125}{70543872}$	$-\frac{875125}{1102248}$	$\frac{5531125}{52907904}$	$-\frac{22905625}{3086944}$	$\frac{39123125}{141087744}$	$-\frac{1707625}{3968028}$
$\tau_{n+\frac{2}{3}}$	1	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{12991}{793800}$	$\frac{2819}{6300}$	$-\frac{2014}{1575}$	$\frac{4757}{2835}$	$-\frac{524}{441}$	$\frac{1117}{2520}$	$-\frac{974}{14175}$
$\tau_{n+\frac{7}{9}}$	1	$\frac{7}{9}$	$\frac{49}{162}$	$\frac{104608483}{4534963200}$	$\frac{167683439}{251942400}$	$-\frac{29630741}{15746400}$	$\frac{93161201}{37791360}$	$-\frac{5486971}{3149280}$	$\frac{65429651}{100776960}$	$-\frac{890771}{8857350}$
$\tau_{n+\frac{8}{9}}$	1	$\frac{8}{9}$	$\frac{32}{81}$	$\frac{6711872}{217005075}$	$\frac{3197696}{3444525}$	$-\frac{8966144}{3444525}$	$\frac{7043072}{2066715}$	$-\frac{2318336}{964467}$	$\frac{616768}{688905}$	$-\frac{4296704}{31000725}$
τ_{n+1}	1	1	$\frac{1}{2}$	$\frac{50117}{1254400}$	$\frac{110727}{89600}$	$-\frac{1377}{400}$	$\frac{20187}{4480}$	$-\frac{24867}{7840}$	$\frac{42363}{35840}$	$-\frac{3071}{16800}$

Table 2. Coefficient of ϑ'_j s and ϕ'_j s for the second scheme of (5) which evaluated at all points as

τ_n	y'_n	y''_n	f_n	$f_{n+\frac{4}{9}}$	$f_{n+\frac{5}{9}}$	$f_{n+\frac{2}{3}}$	$f_{n+\frac{7}{9}}$	$f_{n+\frac{8}{9}}$	f_{n+1}
$\tau_{n+\frac{4}{9}}$	1	$\frac{4}{9}$	$\frac{54962}{1607445}$	$\frac{4892}{5103}$	$-\frac{7808}{2835}$	$\frac{276352}{76545}$	$-\frac{456448}{178605}$	$\frac{386}{405}$	$-\frac{6784}{45927}$
$\tau_{n+\frac{5}{9}}$	1	$\frac{5}{9}$	$\frac{7348025}{164602368}$	$\frac{197375}{145152}$	$-\frac{1250825}{326592}$	$\frac{4896875}{979776}$	$-\frac{149375}{42336}$	$\frac{3431875}{2612736}$	$-\frac{85375}{419904}$
$\tau_{n+\frac{2}{3}}$	1	$\frac{2}{3}$	$\frac{4373}{79380}$	$\frac{37}{21}$	$-\frac{514}{105}$	$\frac{1207}{189}$	$-\frac{3308}{735}$	$\frac{703}{420}$	$-\frac{734}{2835}$
$\tau_{n+\frac{7}{9}}$	1	$\frac{7}{9}$	$\frac{220157}{3359232}$	$\frac{2019241}{933120}$	$-\frac{463393}{77760}$	$\frac{5447869}{699840}$	$-\frac{127645}{23328}$	$\frac{1265327}{622080}$	$-\frac{132055}{419904}$
$\tau_{n+\frac{8}{9}}$	1	$\frac{8}{9}$	$\frac{122144}{1607445}$	$\frac{21824}{8505}$	$-\frac{5120}{729}$	$\frac{702976}{76545}$	$-\frac{382976}{59535}$	$\frac{12224}{5103}$	$-\frac{84992}{229635}$
τ_{n+1}	1	1	$\frac{32527}{376320}$	$\frac{5319}{1792}$	$-\frac{18117}{2240}$	$\frac{3387}{320}$	$-\frac{57969}{7840}$	$\frac{49599}{17920}$	$-\frac{571}{1344}$

Table 3. Coefficient of $\vartheta''_j\, s$ and $\phi''_j\, s$ for the third scheme of (5) which evaluated at all points as

τ_n	y''_n	f_n	$f_{n+\frac{4}{9}}$	$f_{n+\frac{5}{9}}$	$f_{n+\frac{2}{3}}$	$f_{n+\frac{7}{9}}$	$f_{n+\frac{8}{9}}$	f_{n+1}
$\tau_{n+\frac{4}{9}}$	1	$\frac{16798}{178605}$	$\frac{10154}{2835}$	$-\frac{27568}{2835}$	$\frac{106424}{8505}$	$-\frac{173984}{19845}$	$\frac{9208}{2835}$	$-\frac{12784}{25515}$
$\tau_{n+\frac{5}{9}}$	1	$\frac{859885}{9144576}$	$\frac{262775}{72576}$	$-\frac{87305}{9072}$	$\frac{678625}{54432}$	$-\frac{277625}{31752}$	$\frac{470375}{145152}$	$-\frac{40825}{81648}$
$\tau_{n+\frac{2}{3}}$	1	$\frac{9953}{105840}$	$\frac{1013}{280}$	$-\frac{67}{7}$	$\frac{7901}{630}$	$-\frac{2146}{245}$	$\frac{1817}{560}$	$-\frac{473}{945}$
$\tau_{n+\frac{7}{9}}$	1	$\frac{17549}{186624}$	$\frac{187621}{51840}$	$-\frac{62083}{6480}$	$\frac{490147}{38880}$	$-\frac{28189}{3240}$	$\frac{335797}{103680}$	$-\frac{5831}{11664}$
$\tau_{n+\frac{8}{9}}$	1	$\frac{16796}{178605}$	$\frac{10256}{2835}$	$-\frac{27136}{2835}$	$\frac{107008}{8505}$	$-\frac{171008}{19845}$	$\frac{9316}{2835}$	$-\frac{2560}{5103}$
τ_{n+1}	1	$\frac{5897}{62720}$	$\frac{16227}{4480}$	$-\frac{5373}{560}$	$\frac{14151}{1120}$	$-\frac{17037}{1960}$	$\frac{30483}{8960}$	$-\frac{261}{560}$

Numerical Conditions that Satisfied the Properties

Here the properties to confirm the convergence of the method will examine. It is known that for a scheme to converge, the consistency and zero-stability must be fulfilled (Lambert 1973).

we use the linear operator $L\{y(x):h\}$ as

$$L\{y(x):h\} = A^{(0)}Y_m^{(i)} - \sum_{i=0}^{k}\frac{jh^{(i)}}{i!}y_n^{(i)} - h^{(3-1)}\left[d_i f(y_n) + b_i F(Y_m)\right] \quad (6)$$

To find the order and error constant of the method, the Taylor series of Y_m *and* $F(Y_m)$ is expanded on (6) and relate the terms of h to yield

$$L\{y(x):h\} = C_0 y(x) + C_1 y'(x) + \cdots + C_p h^p y^p(x) + C_{p+1}h^{p+1}y^{p+1}(x) + C_{p+2}h^{p+2}y^{p+2}(x) + \cdots \quad (7)$$

Definition 3.1: L is called the linear operator, which is associate with block method are supposed to be of order p if $C_0 = C_1 = \cdots = C_p = C_{p+1} = C_{p+2} = 0, \quad C_{p+3} \neq 0.$ C_{p+3} is called the error constant and implies that the truncation error is given by $t_{n+k} = C_{p+3}h^{p+3}y^{p+3}(x) + 0h^{p+4}$

The operator is then expanded using Taylor series to yield

$$\left[\begin{array}{l}
\sum_{j=0}^{\infty}\frac{\left(\frac{4}{9}\right)^{j}}{j!}-y_n-\frac{4}{9}hy'_n-\frac{4}{81}h^2y''_n-\frac{1398676}{217005075}h^3y'''_n-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{j+3}\left[\begin{array}{l}-\frac{500368}{3444525}\left(\frac{4}{9}\right)+\frac{1474432}{3444525}\left(\frac{5}{9}\right)-\frac{167488}{295245}\left(\frac{2}{3}\right)+\frac{1948928}{4822335}\left(\frac{7}{9}\right)\\+\frac{104276}{688905}\left(\frac{8}{9}\right)-\frac{729472}{31000725}(1)\end{array}\right]\\
\sum_{j=0}^{\infty}\frac{\left(\frac{5}{9}\right)^{j}}{j!}-y_n-\frac{5}{9}hy'_n-\frac{25}{162}h^2y''_n-\frac{96218375}{8888527872}h^3y'''_n-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{j+3}\left[\begin{array}{l}-\frac{19331125}{70543872}\left(\frac{4}{9}\right)+\frac{875125}{1102248}\left(\frac{5}{9}\right)-\frac{55319375}{52907904}\left(\frac{2}{3}\right)+\\\frac{22905625}{30862944}\left(\frac{7}{9}\right)+\frac{39123125}{141087744}\left(\frac{8}{9}\right)-\frac{1707625}{39680928}(1)\end{array}\right]\\
\sum_{j=0}^{\infty}\frac{\left(\frac{2}{3}\right)^{j}}{j!}-y_n-\frac{2}{3}hy'_n-\frac{2}{9}h^2y''_n-\frac{12991}{793800}h^3y'''_n-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{j+3}\left[-\frac{2819}{6300}\left(\frac{4}{9}\right)+\frac{2014}{1575}\left(\frac{5}{9}\right)-\frac{4757}{2835}\left(\frac{2}{3}\right)+\frac{524}{441}\left(\frac{7}{9}\right)+\frac{1117}{2520}\left(\frac{8}{9}\right)-\frac{974}{14175}(1)\right]\\
\sum_{j=0}^{\infty}\frac{\left(\frac{7}{9}\right)^{j}}{j!}-y_n-\frac{7}{9}hy'_n-\frac{49}{162}h^2y''_n-\frac{104608483}{4534963200}h^3y'''_n-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{j+3}\left[\begin{array}{l}-\frac{167683439}{251942400}\left(\frac{4}{9}\right)+\frac{29630741}{15746400}\left(\frac{5}{9}\right)-\frac{93161201}{37791360}\left(\frac{2}{3}\right)+\frac{5486971}{3149280}\left(\frac{7}{9}\right)\\+\frac{65429651}{100776960}\left(\frac{8}{9}\right)-\frac{890771}{8857350}(1)\end{array}\right]\\
\sum_{j=0}^{\infty}\frac{\left(\frac{8}{9}\right)^{j}}{j!}-y_n-\frac{8}{9}hy'_n-\frac{32}{81}h^2y''_n-\frac{6711872}{217005075}h^3y'''_n-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{j+3}\left[\begin{array}{l}-\frac{3197696}{3444525}\left(\frac{4}{9}\right)+\frac{8966144}{3444525}\left(\frac{5}{9}\right)-\frac{7043072}{2066715}\left(\frac{2}{3}\right)+\frac{2318336}{964467}\left(\frac{7}{9}\right)\\+\frac{616768}{688905}\left(\frac{8}{9}\right)-\frac{4296704}{31000725}(1)\end{array}\right]\\
\sum_{j=0}^{\infty}\frac{(1)^{j}}{j!}-y_n-hy'_n-\frac{1}{2}h^2y''_n-\frac{50117}{1254400}h^3y'''_n-\sum_{j=0}^{\infty}\frac{h^{j+3}}{j!}y_n^{j+3}\left[-\frac{110727}{89600}\left(\frac{4}{9}\right)+\frac{1377}{400}\left(\frac{5}{9}\right)-\frac{20187}{4480}\left(\frac{2}{3}\right)+\frac{24867}{7840}\left(\frac{7}{9}\right)+\frac{42363}{35840}\left(\frac{8}{9}\right)-\frac{3011}{16800}(1)\right]
\end{array}\right]$$

the coefficient of h is compared, according to (Raymond, Skwame & Lydia 2021) the method is found to uniformly order six, while the error constant are found individually as

$$\left[1.7180\times10^{-7},\quad 1.7159\times10^{-7},\quad 1.7167\times10^{-7},\quad 1.7160\times10^{-7},\quad 1.7171\times10^{-7},\quad 1.7131\times10^{-7}\right]$$

Traditionally, the method is consistent because the order of the method is order greater than or equal to one.
By definition, the method is said to be zero stable as $h \to 0$ if the roots of the polynomial $\pi(r)=0$ satisfy $\left|\left[\sum A^0R^{k-1}\right]\right|\leq 1$, and those roots with R = 1 must be simple.

Hence according to [8] it's found as

$$\pi(r)=\left|r\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}\right|=\left|\begin{bmatrix}r&0&0&0&0&-1\\0&r&0&0&0&-1\\0&0&r&0&0&-1\\0&0&0&r&0&-1\\0&0&0&0&r&-1\\0&0&0&0&0&r-1\end{bmatrix}\right|=r^6(r-1)$$

Then, solving for r in $r^6(r-1)$,

gives $r=0,\,0,0,0,\,0,1$. Therefore, the method is zero stable.

According to the theorem of Dahlquist, the scheme is convergent, for the consistency and zero-stability are analyzed and fulfilled (Dahlquist 1959).

Mathematical computation of the method

The following sampled models are carefully use for presenting the accurateness of the scheme with the previous schemes of (Kuboye & Omar 2016, Areo & Omojola 2017, Adeyeye & Omar 2018, Fasasi 2018, Omar, Abdullahi & Kuboye 2016, Adesanya, Udoh & Ajileye 2013).

The following notations will be used in the table below.

KO16 means error in Kuboye & Omar 2016

AO17 means error in Areo & Omojola 2017

AO18 means error in Adeyeye & Omar 2018

F18 means error in Fasasi 2018

OAK16 means error in Omar, Abdullahi & Kuboye 2016

AUA13 means error in Adesanya, Udoh & Ajileye 2013

Sample one is the highly non-stiff third order problematic differential equation considered by (Kuboye & Omar 2016, Areo & Omojola 2017, Adeyeye & Omar 2018, Fasasi 2018)

$$y''' = \exp(t),\ y(0) = 3,\ y'(0) = 1,\ y''(0) = 5,\ h = 0.1$$

while the actual result is

$$y(t) = 2(1 + t^2) + e^t$$

Table one present the outcomes for solving sampled problem one

t	Actual Result	Calculated Result	Error in our method	AO17	KO16	AO18	F18
0.1	3.12517091807564762480	3.12517091807564761800	6.8000e-18	2.6645e-15	2.2205e-15	6.3427e-13	4.4090e-16
0.2	3.30140275816016983390	3.30140275816016979490	3.9000e-17	4.4409e-16	1.4211e-14	2.3288e-12	8.8818e-16
0.3	3.52985880757600310400	3.52985880757600298670	1.1730e-16	3.1086e-15	3.6415e-14	5.4435e-12	2.2205e-15
0.4	3.81182469764127031780	3.81182469764127005280	2.6500e-16	6.6613e-15	6.8390e-14	9.8532e-12	3.5527e-15
0.5	4.14872127070012814680	4.14872127070012763960	5.0720e-16	9.7697e-15	1.0925e-13	1.5997e-11	5.3251e-15
0.6	4.54211880039050897490	4.54211880039050810260	8.7230e-16	2.0428e-14	1.6076e-13	2.3722e-11	7.9936e-15
0.7	4.99375270747047652160	4.99375270747047513080	1.3908e-15	2.1316e-14	2.2116e-13	3.3568e-11	1.3323e-14
0.8	5.50554092849246760460	5.50554092849246550720	2.0974e-15	1.8652e-14	2.9221e-13	4.5344e-11	1.9540e-14
0.9	6.07960311115694966380	6.07960311115694663400	3.0298e-15	2.2205e-14	3.6948e-13	5.9708e-11	2.7534e-14
1.0	6.71828182845904523540	6.71828182845904100560	4.2298e-15	2.1316e-14	4.6718e-13	7.6432e-11	3.3571e-14

Basis [AO17, KO16, AO18 and F18].

Sample two is the highly non-stiff third order problematic differential equation considered by (Areo & Omojola 2017, Adeyeye & Omar 2018, Omar, Abdullahi & Kuboye 2016).

$$y''' = y,\ y(0) = 1,\ y'(0) = -1,\ y''(0) = 1,\ h = 0.1$$

while the actual result is

$$y(t) = \exp(-t)$$

Table two present the outcomes for solving sample two

t	Actual Result	Computed Result	Error in our Method	OAK16	AO18	AO16
0.1	0.90483741803595957316	0.90483741803595956712	6.0400e-18	3.5442e-11	5.2181e-13	6.0396e-14
0.2	0.81873075307798185867	0.81873075307798182512	3.3550e-17	5.3218e-11	1.9342e-12	4.7196e-13
0.3	0.74081822068171786607	0.74081822068171776941	9.6660e-17	1.3712e-09	4.3127e-12	1.5618e-12
0.4	0.67032004603563930074	0.67032004603563909264	2.0810e-16	8.4438e-09	7.4462e-12	3.6204e-12
0.5	0.60653065971263342360	0.60653065971263304428	3.7932e-16	2.9737e-08	1.1436e-11	6.9109e-12
0.6	0.54881163609402643263	0.54881163609402581209	6.2054e-16	7.8367e-08	1.6124e-11	1.6667e-11
0.7	0.49658530379140951470	0.49658530379140857399	9.4071e-16	4.3298e-05	2.1614e-11	1.8098e-11
0.8	0.44932896411722159143	0.44932896411722024376	1.3477e-15	9.6716e-05	2.7797e-11	2.6384e-11
0.9	0.40656965974059911188	0.40656965974059726386	1.8480e-15	1.6111e-04	3.4785e-11	3.6683e-11
1.0	0.36787944117144232160	0.36787944117143987439	2.4472e-15	2.3725e-04	4.2511e-11	4.9122e-11

Basic [OAK16, AO18 and AO16].

Sample three is the homogeneous third order problematic differential equation considered by (Areo & Omojola 2017, Adesanya, Udoh & Ajileye 2013).

$$y''' = y',\ y(0) = 0,\ y'(0) = 1,\ y''(0) = 2,\ h = 0.1$$

while the actual result is

$$y(t) = 2(1 - \cos t) + \sin t$$

Table three present the outcomes for solving sample three

g	Actual Result	Computed Result	Error in our Method	AUJ13	AO17
0.1	0.10982508609077662011	0.10980685250209750181	1.8234e-05	1.6613e-12	1.1177e-10
0.2	0.23853617511257795326	0.23842932363423049866	1.0685e-04	7.5411e-12	9.3348e-10
0.3	0.38484722841012753581	0.38484722841013871413	1.1178e-14	1.3843e-09	3.2775e-09
0.4	0.54729635430288032607	0.54729635430288058082	2.5475e-16	4.5006e-09	8.0524e-09
0.5	0.72426041482345756807	0.72426041482345808858	5.2051e-16	1.0520e-08	1.6249e-08
0.6	0.91397124357567876270	0.91397124357567967928	9.1658e-16	1.9715e-08	2.8912e-08
0.7	1.11453331266871420120	1.11453331266871566160	1.4604e-15	3.2968e-08	4.7125e-08
0.8	1.32394267220519191980	1.32394267220519408470	2.1649e-15	5.0419e-08	7.1985e-08
0.9	1.54010697308615447550	1.54010697308615751340	3.0379e-15	7.2608e-08	1.0458e-07
1.0	1.76086637307161707180	1.76086637307162115380	4.0820e-15	9.9511e-08	1.4596e-07

Basic [AUJ13 and AO17].

Summary and Conclusion

The effective algorithm on mathematical treatment of order three problematic systems was established and examined in this research. On the process of examining the conditions that satisfied the properties of the method, we have discovered that all the conditions are valid. The highly non-stiff and homogeneous third order problematic systems of differential equation are directly solved and relate with the method developed by (Kuboye & Omar 2016, Areo & Omojola 2017, Adeyeye & Omar 2018, Fasasi 2018, Omar, Abdullahi & Kuboye 2016, Adesanya, Udoh & Ajileye 2013) in this research, obvious result shows better convergence than that of (Kuboye & Omar 2016, Areo & Omojola 2017, Adeyeye & Omar 2018, Fasasi 2018, Omar, Abdullahi & Kuboye 2016, Adesanya, Udoh & Ajileye 2013) in literature. Hence direct solution of higher order problematic equation has advantages over reducing (1) to equation of order one.

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Competing Interests
Authors have declared that no competing interests exist.

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